

# Geometric reduction of spin trigonometric RS systems

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# Plan for the talk

- 1 **Motivation : CM system**
- 2 RS system – what is known
- 3 RS system – new real form

# Motivation (1) : CM system

[Calogero,'71 – Sutherland,'71 – Moser,'75]

Calogero-Moser-Sutherland (CM) system with trigonometric potential:

$$\ddot{q}_i = \gamma^2 \sum_{j \neq i} \frac{\cot\left(\frac{1}{2}(q_i - q_j)\right)}{\sin^2\left(\frac{1}{2}(q_i - q_j)\right)}, \quad \text{for } i = 1, \dots, n$$

(parameter  $\gamma \neq 0$ , real or complex. 'Positions'  $e^{iq_j}$  on  $S^1$  or  $\mathbb{C}^\times$ )

Definition as an integrable Hamiltonian system:

$$H_{CM} = \frac{1}{2} \sum_{1 \leq i \leq n} p_i^2 + \sum_{1 \leq i < j \leq n} \frac{\gamma^2}{\sin^2\left(\frac{1}{2}(q_i - q_j)\right)}$$

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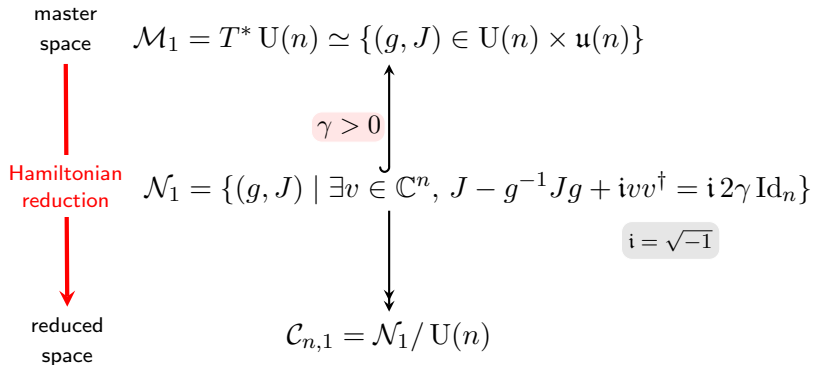
Definition as an integrable Hamiltonian system:

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**Question:** What is the phase space?

# Motivation (2) : CM system by reduction

Approach\* from [Kazhdan,Kostant,Sternberg;'78]



Integrable system from  $\text{tr}(J), \dots, \text{tr}(J^n)$

\* Real case. Holomorphic case: replace  $\mathfrak{u}(n), U(n)$  by  $\mathfrak{gl}_n(\mathbb{C}), \text{GL}_n(\mathbb{C})$  [Wilson,'98]

## Motivation (3) : spin CM system

CM system with “vector” spins based on [Gibbons-Hermsen,'84]

coordinates\*:  $(q_i, p_i, a_i^\alpha, b_i^\alpha)_{\substack{1 \leq \alpha \leq d \\ 1 \leq i \leq n}}$  with  $2n$  constraints  $\rightsquigarrow \dim = 2nd$

$$\ddot{q}_i = \sum_{j \neq i} f_{ij} f_{ji} \frac{\cot\left(\frac{1}{2}(q_i - q_j)\right)}{4 \sin^2\left(\frac{1}{2}(q_i - q_j)\right)} \quad (\text{for } i = 1, \dots, n \text{ and } 1 \leq \alpha \leq d)$$

$$\dot{a}_i^\alpha = \sum_{j \neq i} \frac{f_{ij}}{4 \sin^2\left(\frac{1}{2}(q_i - q_j)\right)} a_j^\alpha, \quad \dot{b}_i^\alpha = - \sum_{j \neq i} \frac{f_{ji}}{4 \sin^2\left(\frac{1}{2}(q_i - q_j)\right)} b_j^\alpha$$

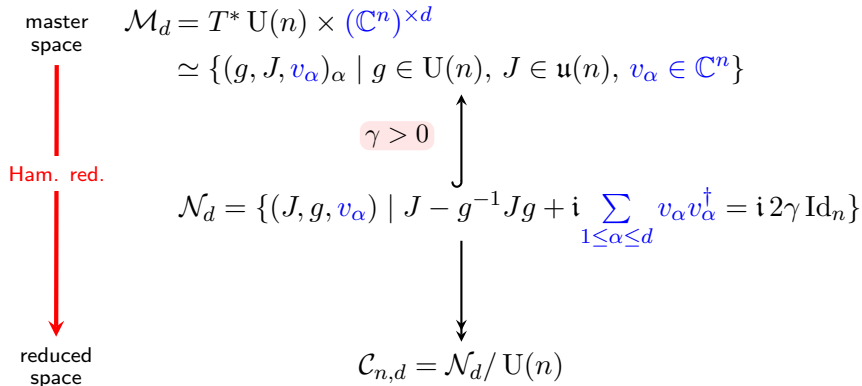
(We set  $f_{ij} = \sum_{1 \leq \alpha \leq d} a_i^\alpha b_j^\alpha$ . Constraints include  $f_{jj} = 2\gamma$ ,  $\gamma \neq 0$ .)

$$H_{CM}^{spin} = \frac{1}{2} \sum_{1 \leq i \leq n} p_i^2 + \sum_{1 \leq i < j \leq n} \frac{f_{ij} f_{ji}}{4 \sin^2\left(\frac{1}{2}(q_i - q_j)\right)}$$

\*For the real case:  $b_i^\alpha = \bar{a}_i^\alpha \in \mathbb{C}$  + reality of Poisson bracket.

# Motivation (4) : spin CM system by reduction

Same idea as “without spins” (which corresponds to  $d = 1$ )



**Degenerate integrability:** algebra of first integrals of  $(\mathrm{tr}(J^k))_k$  of codim.  $n$

Complex case:  $\mathfrak{u}(n), \mathrm{U}(n) \rightsquigarrow \mathfrak{gl}_n(\mathbb{C}), \mathrm{GL}_n(\mathbb{C})$ , and  $(v, v^\dagger) \rightsquigarrow (v, w) \in T^*\mathbb{C}^n$ .

Note: For “coadjoint orbit”-valued spins, see e.g. L. Fehér, L.-C. Li, N. Reshetikhin, ...

## Motivation (5) : RS system

[Ruijsenaars-Schneider, '86] : “relativistic” version of CM system.

Ruijsenaars-Schneider (RS) system with trigonometric potential

$$H_{RS} = \sum_{1 \leq i \leq n} e^{p_i} \prod_{k \neq i} \frac{\sin(q_i - q_k + \gamma)}{\sin(q_i - q_k)}$$

(“MacDonald form” of the complex Hamiltonian.  $\gamma \neq 0$ .)

**Question:** What are the analogues of the previous constructions?



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# Trigonometric RS system : 2 important approaches

Trig. CM (no spins): Hamiltonian reduction of  $T^*G$  by  $G$ ,  $G = U(n), GL_n(\mathbb{C})$

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Approaches for the trig. RS system:

	complex case	real case
Hamiltonian reduction (Poisson-Lie)	[Fock-Rosly, '99]	[Fehér-Klimcik, '11] (non-compact)
quasi-Hamiltonian reduction	[Oblomkov, '04] [Chalykh-F., '17]	[Fehér-Klimcik, '12] (compact)

Note: reduction starts with a *finite-dimensional* master space

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**We will outline the 'quasi-' case**

Note: reduction starts with a *finite-dimensional* master space

# General construction

( $G = U(n)$  or  $GL_n(\mathbb{C})$  if real/complex.)

**quasi-Poisson point of view** [Alekseev–Kosmann-Schwarzbach–Meinrenken, '02]  
(+ [Alekseev–Malkin–Meinrenken, '98]; [Boalch, '07]; ... )

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$\mathcal{M}^{qP}$ : (internally fused) quasi-Poisson double

$$\mathcal{M}^{qP} \simeq G \times G$$

- quasi-Poisson bracket
  - $G \curvearrowright$  simult. conjugation
  - action by quasi-Poisson morphisms
  - Group-valued moment map  $\Phi : \mathcal{M}^{qP} \rightarrow G$
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$\mathcal{N} = \Phi^{-1}(C)$  minimal smooth subspace of  $\mathcal{M}^{qP}$

( $\rightsquigarrow C = q \text{Id}_n + \text{rank}1$  for  $q = e^{i\gamma}$  or  $q \in \mathbb{C}^\times$  generic)

$$\mathcal{C}_n = \mathcal{N}/G \text{ smooth symplectic of dim. } 2n$$

$\rightsquigarrow \exists$  local coordinates  $(q_i, p_i)_{i=1}^n$  giving interpretation as trig. RS system

# Dynamics

If  $(A, B) \in G \times G \simeq \mathcal{M}^{qP}$ , consider

$$\mathfrak{H} := \{H \mid H(A, B) = h(A), h \in \mathcal{C}^\infty(\mathrm{U}(n))^{\mathrm{U}(n)}\}$$

$\rightsquigarrow \mathfrak{H}$  defines an integrable system on  $\mathcal{C}_n = \mathcal{N}/G$

**Byproduct of the reduction:** get *explicit* commuting flows on  $\mathcal{M}^{qP}$ .

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**Byproduct of the reduction:** get *explicit* commuting flows on  $\mathcal{M}^{qP}$ .

Example (Real case)  $(A, B) \in \mathrm{U}(n) \times \mathrm{U}(n)$

$$H_1 = \Re[\mathrm{tr}(A)], \dot{A} = 0, \dot{B} = \frac{1}{2}B(A - A^{-1})$$

$$H_2 = \Im[\mathrm{tr}(A)], \dot{A} = 0, \dot{B} = -\frac{i}{2}B(A + A^{-1})$$

$$\forall H \in \mathfrak{H}, \dot{A} = 0, \dot{B} = -B\nabla h(A)$$

$$\Rightarrow A(t) = A_0, B(t) = B_0 \exp(-t\nabla h(A_0)) \rightsquigarrow \text{complete flows}$$

# Spin trigo. RS system

Trigo. RS system with *spins* in the complex setting [Krichever-Zabrodin,'95]

coordinates:  $(q_i, a_i^\alpha, c_i^\alpha)_{\substack{1 \leq \alpha \leq d \\ 1 \leq i \leq n}}$  with  $n$  constraints  $\rightsquigarrow \dim = 2nd$

$$\ddot{q}_i = \sum_{j \neq i} f_{ij} f_{ji} (V(q_i - q_j) - V(q_j - q_i)) , \quad \text{for } i = 1, \dots, n$$

$$\dot{a}_i^\alpha = \sum_{j \neq i} f_{ij} V(q_i - q_j) a_j^\alpha , \quad \dot{c}_i^\alpha = - \sum_{j \neq i} f_{ji} V(q_j - q_i) c_j^\alpha , \quad 1 \leq \alpha \leq d$$

(We set  $f_{ij} = \sum_{1 \leq \alpha \leq d} a_i^\alpha c_j^\alpha$ ,  $V(q) = \coth(q) - \coth(q + \gamma)$ ,  $\gamma \neq 0$ .)

**Question 1:** What is the Hamiltonian formulation?

(Partial conjecture in [Arutyunov-Frolov,'98].)

**Question 2:** What is (are) the real form(s)?

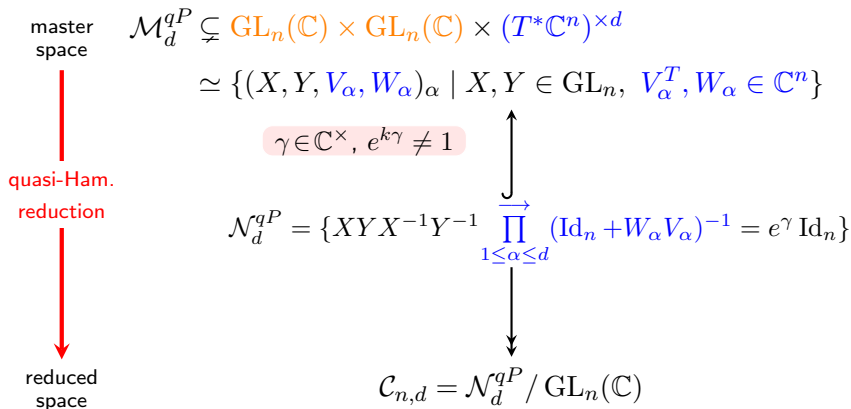


# spin RS systems : 2 important approaches

	complex case	real case
Hamiltonian reduction (Poisson-Lie)	[Arutyunov-Olivucci, '20] (2)	[F.-Fehér-Marshall, '21] (3) (non-compact)
quasi-Hamiltonian reduction	[Chalykh-F., '20] (1)	[F.-Fehér, '23] (4) (non-compact)

Today: continue focus on the quasi-Hamiltonian point of view

# spin RS systems : quasi-Hamiltonian red. / $\mathbb{C}$ (1)



Quasi-Poisson structure from  $\{(V, W) \in T^*\mathbb{C}^n \mid \det(\text{Id}_n + WV) \neq 0\} \rightsquigarrow$  [Van den Bergh, '08].

# spin RS systems : quasi-Hamiltonian red. / $\mathbb{C}$ (2)

## Theorem ([Chalykh-F., '20])

*On open subset of  $\mathcal{C}_{n,d}$ , there exist local coordinates so that*

- equations of motion associated with Hamiltonian  $\text{tr}(Y)$  reproduce the equations of the spin trig. RS system of [Krichever-Zabrodin, '95];*
- the Poisson bracket written in local coordinates allows to prove the conjecture of [Arutyunov-Frolov, '98].*

Flows before reduction:

$$\dot{X} = XY, \quad \dot{Y} = 0, \quad \dot{V}_\alpha = 0, \quad \dot{W}_\alpha = 0$$

Moreover: integrability (Liouville + degenerate)

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# The missing piece $/\mathbb{R}$ (1)

From now on: following [F.-Fehér, arXiv:2302.14392]

**Objective:** quasi-Hamiltonian reduction from master space

$$\begin{aligned}\mathcal{M}_d^{qP} &= \mathbf{U}(n) \times \mathbf{U}(n) \times [???]^{\times d} \\ &\subsetneq \mathbf{U}(n) \times \mathbf{U}(n) \times (\mathbb{C}^n)^{\times d}\end{aligned}$$

complex case:

$$\begin{aligned}\mathcal{M}_d^{qP} &= \mathrm{GL}_n(\mathbb{C}) \times \mathrm{GL}_n(\mathbb{C}) \times (\{(V, W) \mid 1 + VW \neq 0\})^{\times d} \\ &\subsetneq \mathrm{GL}_n(\mathbb{C}) \times \mathrm{GL}_n(\mathbb{C}) \times (T^*\mathbb{C}^n)^{\times d}\end{aligned}$$

## The missing piece / $\mathbb{R}$ (2)

For  $x \in \mathbb{R} \setminus \{0\}$ , block [??] is  $D(x) := \{v \in \mathbb{C}^n \mid |v|^2 < \frac{2\pi}{|x|}\} \subset \mathbb{C}^n \simeq T^*\mathbb{R}^n$

**Theorem** ([F.-Fehér,'23])

*The following defines a quasi-Poisson bracket on  $D(x)$ :*

$$\{v_i, v_k\} = 0, \quad \{\bar{v}_i, \bar{v}_k\} = 0, \quad \{v_i, \bar{v}_k\} = \frac{i}{x} \delta_{ik} + \frac{i}{2} \mathbf{b}(x|v|^2) [ |v|^2 \delta_{ik} - v_i \bar{v}_k ],$$

*for left mult. of  $U(n)$  and  $\mathbf{b}(t) = \cot\left(\frac{t}{2}\right) - \frac{2}{t}$ .*

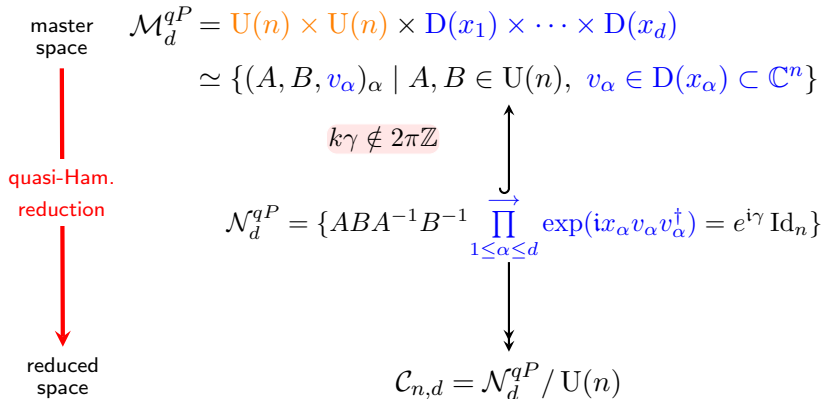
Key remark:  $\Phi : D(x) \rightarrow U(n)$ ,  $\Phi(v) = \exp(\mathbf{i}xvv^\dagger)$  is moment map.

**Two proofs:**

$\rightsquigarrow$  directly from the definition

$\rightsquigarrow$  *exponentiation* of  $(T^*\mathbb{R}^n, \{-, -\}_{\text{can}})$  (in sense of [AKSM,'02]);  
corresponds to 2-form of [Hurtubise-Jeffrey-Sjamaar,'06]

# spin RS systems : quasi-Hamiltonian red. / $\mathbb{R}$ (1)



To construct: local coordinates for interpretation as real spin RS system

$\rightsquigarrow$  Complex approach of [Chalykh-F.,20] not working!

## spin RS systems : quasi-Hamiltonian red. / $\mathbb{R}$ (2)

$$\mathcal{C}_{n,d} = \{ABA^{-1}B^{-1} \prod_{1 \leq \alpha \leq d}^{\rightarrow} \exp(ix_{\alpha} v_{\alpha} v_{\alpha}^{\dagger}) = e^{i\gamma} \text{Id}_n\} / \text{U}(n)$$

---

### Theorem ([F.-Fehér,'23])

- $\mathcal{C}_{n,d}$  not compact (*for*  $d > 1$ )
- $\mathcal{C}_{n,d}$  is smooth when  $e^{ik\gamma} \neq 1$  for  $k = 1, \dots, n$
- On a  $n$ -dimensional cover of  $\mathcal{C}_{n,d}$ , the vector fields induced by the functions  $\Re[\text{tr}(A)]$ ,  $\Im[\text{tr}(A)]$  can be combined (complex-linearly) to give equations of motion of the spin RS system of [Krichever-Zabrodin,'95]



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What about integrability of  $\mathfrak{H} := \{h(A) \mid h \in \mathcal{C}^{\infty}(\text{U}(n))\}^{\text{U}(n)}$  ???


# Dynamics on the master phase space

$$\mathcal{M}_d := \mathcal{M}_d^{qP} = \mathrm{U}(n) \times \mathrm{U}(n) \times \mathrm{D}(x_1) \times \cdots \times \mathrm{D}(x_d) \ni (A, B, v_\alpha)$$

$\rightsquigarrow$  **Hamiltonian quasi-Poisson manifold**<sup>1</sup> constructed by fusion of

- internally fused double  $\mathbb{D}(\mathrm{U}(n)) \simeq \mathrm{U}(n) \times \mathrm{U}(n)$
- $d$  Hamiltonian quasi-Poisson balls  $\mathrm{D}(x_\alpha)$

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<sup>1</sup>In fact,  $\exists$  pencil of compatible such structures; all are quasi-Hamiltonian! 

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**Theorem** ([F.-Fehér, '23])

Set  $\mathfrak{H} := \{H \mid H(A, B, v_\alpha) = h(A), h \in \mathcal{C}^\infty(\mathrm{U}(n))^{\mathrm{U}(n)}\}$ .

$\forall H \in \mathfrak{H}$ , the quasi-Hamiltonian vector field  $X_H$  preserves the  $\mathrm{U}(n)$ -equivariant smooth map  $\Psi : \mathcal{M}_d \rightarrow \mathcal{M}_d$  defined by

$$\Psi : (A, B, v_1, \dots, v_d) \mapsto (A, BAB^{-1}, v_1, \dots, v_d)$$

Moreover  $\mathfrak{F} := \Psi^*(\mathcal{C}^\infty(\mathcal{M}_d))$  is a ring of first integrals with functional dimension  $\dim(\mathcal{M}_d) - n \Rightarrow (\mathfrak{H}, \mathfrak{F})$  is a “degenerate integrable system”

<sup>1</sup>In fact,  $\exists$  pencil of compatible such structures; all are quasi-Hamiltonian! 

# Dynamics on the reduced Poisson space (1)

The “interesting” slice  $\mathcal{C}_{n,d} = \Phi^{-1}(e^{i\gamma} \text{Id}_n) / \text{U}(n)$  lies inside  $\mathcal{M}_d / \text{U}(n)$

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Let  $\mathcal{M}_{d*} \subset \mathcal{M}_d$  where  $\text{U}(n)$  acts freely. Set  $\mathcal{M}_{d*}^{\text{red}} := \mathcal{M}_{d*} / \text{U}(n)$   
( $\mathcal{M}_{d*}^{\text{red}}$  filled by symplectic leaves such as  $\mathcal{C}_{n,d}$ , but generic codimension is  $n$ )

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Let  $\mathcal{M}_{d**} \subset \mathcal{M}_{d*}$  where (i)  $\text{U}(n)$  acts freely on  $(A, v_\alpha)$ ; (ii)  $A \in \text{U}(n)_{\text{reg}}$   
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Theorem ([F.-Fehér, '23])

$\forall H \in \mathfrak{H}$ , the quasi-Hamiltonian vector field  $X_H$  restricted to  $\mathcal{M}_{d**}^{\text{red}}$  preserves the smooth submersion  $\psi_{\text{red}}$  induced on  $\mathcal{M}_{d**}^{\text{red}}$  after restricting  $\Psi : \mathcal{M}_d \rightarrow \mathcal{M}_d$  to  $\mathcal{M}_{d**}$ .

Moreover  $\mathfrak{F}_{\text{red}} := \psi_{\text{red}}^* \mathcal{C}^\infty(\Psi(\mathcal{M}_{d**}) / \text{U}(n))$  is a ring of first integrals with functional dimension  $\dim(\mathcal{M}_{d**}^{\text{red}}) - n$ .

$\Rightarrow (\mathfrak{H}_{\text{red}}, \mathfrak{F}_{\text{red}})$  is a degenerate integrable system on  $\mathcal{M}_{d**}^{\text{red}}$

## Dynamics on the reduced Poisson space (2)

What about restricting this generic integrable system to symplectic leaves?



## Dynamics on the reduced Poisson space (2)

What about restricting this generic integrable system to symplectic leaves?

- 1 This **works** for  $(\Phi^{-1}(C_{\text{reg}}) \cap \mathcal{M}_{d^{**}}) / \text{U}(n) \subset \mathcal{M}_{d^{**}}^{\text{red}}$   
where  $C_{\text{reg}} \subset \text{U}(n)_{\text{reg}}$
- 2 Not yet possible on  $(\Phi^{-1}(e^{i\gamma} \text{Id}_n) \cap \mathcal{M}_{d^{**}}) / \text{U}(n) = \mathcal{C}_{n,d} \cap \mathcal{M}_{d^{**}}^{\text{red}}$

**Thank you for your attention !**

Maxime Fairon

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